MATHS HOLIDAY HOMEWORK (2022-23) Class: XII

ALGEBRA OF MATRICES & DETERMINANTS

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the value of x, y and z if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$. **[Ans.: x = 2, y = 4, z = 3]**

2. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ **[Ans.: a = 1, b = 2, c – 3, d = 4]**

3. If A is a square matrix of order 3 such that |adj A| = 64, then find |A|. **[Ans. 8]**

4. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

5. Write the value of $\begin{vmatrix} \sin 20^o & -\cos 20^o \\ \sin 70^o & \cos 70^o \end{vmatrix}$. **[Ans. 1]**

6. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then write A".

7. If $A^2 - A + I = 0$, then find the inverse of A. **[Ans. I – A]**

8. If A is square matrix of order 3 such that |A| = λ, then write the value of |-A|.

9. If $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find the least positive integral value of k.

10. For what value of x, the following matrix is singular? **[Ans. 3]**

11. If A is non-singular square matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}$, then find $(A^T)^{-1}$.

12. If A and B are non-singular square matrices of the same order, then write the relationship between adj AB, adj A and adj B.

13. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, find the value of |adj A|.

14. If matrix A = [1, 2 , 3], then write AA′, where A′ is the transpose of matrix A. **[Ans. 14]**

15. Evaluate: $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$

16. Find the value of x and y, if

    $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ **[Ans. x = 3, y = 3]**

17. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

18. If A is an invertible matrix of order 3 and |A| = 5, then find |adj A|. **[Ans. 25]**

19. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then find the value of x and y. **[Ans. x = 7, y = 1]**

20. Find the cofactor of a_{12} in the followings:

    $\begin{vmatrix} 1 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ **[Ans. 46]**

21. Find the value of x, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$. **[Ans. x = 1]**

22. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, then find the value of x. **[Ans. 10]**

23. If A is square matrix of order 3 such that |adj A| = 64, then find |A|. **[Ans. ± 8]**

24. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then find the value of y. **[Ans. y = 3]**

25. Write the value of the following determinant:

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$ **[Ans. 0]**

26. If A = [a_{ij}] = $\begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and B = [b_{ij}] = $\begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, then find $a_{22} + b_{21}$. **[Ans. 1]**

27. If $\begin{bmatrix} 7y & 5 \\ 2x-3y & -3 \end{bmatrix} = \begin{bmatrix} -21 & 5 \\ 11 & -3 \end{bmatrix}$, then find the value of x. **[Ans. 1]**

28. If A is invertible matrix of 3 × 3 and |A| = 7, the find $|A^{-1}|$. **[Ans. $\frac{1}{7}$]**

29. Write the value of the following determinant:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

30. Find the value of x form the followings:

$$\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$$ **[Ans. ±2]**

31. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α is A an identity matrix? **[Ans. 0]**

32. What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$? **[Ans. 8]**

33. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k. **[Ans. 17]**

34. If A is a matrix of order 3 × 3, then find $(A^2)^{-1}$.

35. Write the adjoint of the following matrix: $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

36. If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k if |2A| = k |A|. **[Ans. 4]**

37. Write a square matrix of order 2, which is both symmetric and skew symmetric.

38. What positive value of x makes the following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$ **[Ans. 4]**

39. From the following matrix equation, find the value of x:

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$ **[Ans. x = 1, y = 2]**

40. Write A^{-1} for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

41. What is the value of the following determinant?

$$\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$$ **[Ans. 0]**

42. Evaluate the Determinant:

$$\begin{vmatrix} x^2 - x + 1 & x-1 \\ x+1 & x+1 \end{vmatrix}$$

43. Find the minor of the element of second row and third column (a_{23}) in the following determinant:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$ **[Ans. 13]**

44. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, then write the positive value of x.

45. From the following matrix equation, find the value of x:

$\begin{bmatrix}1 & 3\\4 & 5\end{bmatrix}\begin{bmatrix}x\\2\end{bmatrix} = \begin{bmatrix}5\\6\end{bmatrix}$ **[Ans. -1]**

46. If $\Delta = \begin{vmatrix}5 & 3 & 8\\2 & 0 & 1\\1 & 2 & 3\end{vmatrix}$, then write the minor of the element a_{23}. **[Ans. 7]**

47. If $\begin{bmatrix}3 & 4\\2 & x\end{bmatrix}\begin{bmatrix}x\\1\end{bmatrix} = \begin{bmatrix}19\\15\end{bmatrix}$, then find the value of x. **[Ans. 5]**

48. Write the value of the following determinant:

$\begin{vmatrix}102 & 18 & 36\\1 & 3 & 4\\17 & 3 & 6\end{vmatrix}$ **[Ans. 0]**

49. If $\begin{bmatrix}7y & 5\\2x-3y & -3\end{bmatrix} = \begin{bmatrix}-21 & 5\\11 & -3\end{bmatrix}$, then find the value of x. **[Ans. x = 1, y = -3]**

50. If $\begin{vmatrix}x+1 & x-1\\x-3 & x+2\end{vmatrix} = \begin{vmatrix}4 & -1\\1 & 3\end{vmatrix}$, then write the value of x. **[Ans. 2]**

51. Write the order of the product matrix.

$\begin{bmatrix}1\\2\\3\end{bmatrix}[2\ 3\ 4]$.

52. If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix}2 & -3 & 5\\6 & 0 & 4\\1 & 5 & -7\end{vmatrix}$, then write the value of $a_{32}.A_{32}$.

[Ans. 110]

53. For a 2 × 2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the valueof a_{12}. **[Ans. $\frac{1}{2}$]**

54. If $A = \begin{vmatrix}3 & 10\\2 & 7\end{vmatrix}$, then write A^{-1}.

55. If $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$, then write the value of x. **[Ans. 13]**

56. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\\cos\theta & \sin\theta\end{bmatrix}$.

57. Find the value of x + y from the following equation:

$2\begin{bmatrix}x & 5\\7 & y-3\end{bmatrix} + \begin{bmatrix}3 & -4\\1 & 2\end{bmatrix} = \begin{bmatrix}7 & 6\\15 & 14\end{bmatrix}$ **[Ans. 11]**

58. If $A^T = \begin{bmatrix}3 & 4\\-1 & 2\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 2 & 1\\1 & 2 & 3\end{bmatrix}$, then find $A^T - B^T$. **[Ans. $\begin{bmatrix}4 & 3\\-3 & 0\\-1 & -2\end{bmatrix}$]**

59. Find the value of x + y from the following equation:

$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ **[Ans. 6]**

60. If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$, then write the value of x. **[Ans. 3]**

61. Evaluate: $\begin{vmatrix}\cos 15^o & \sin 15^o\\\sin 75^o & \cos 75^o\end{vmatrix}$ **[Ans. 0]**

62. For what value of x, is the matrix $A = \begin{bmatrix}0 & 1 & -2\\-1 & 0 & 3\\x & -3 & 0\end{bmatrix}$ a skew-symmetric matrix? **[Ans. 2]**

63. If matrix $A = \begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}$ and $A^2 = kA$, then write the value of k. **[Ans. 2]**

64. If A is a 3 × 3 matrix, whose elements are given by $a_{ij} = \frac{1}{3}|-3i + j|$, then write the value of a_{23}. **[Ans. 1]**

65. If A is a square matrix and |A| = 2, then write the value of |AA′|, where A′ is the transpose of matrix A.

SHORT ANSWER TYPE QUESTIONS [4 MARKS]

1. Find the value of x, if $[1 \;\; x \;\; 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\2\\x\end{bmatrix} = 0$

2. If $A = \begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then find the value of λ for which A^{-1} exists. **[Ans. $-\frac{8}{5}$]**

3. Using properties of determinant, prove that

$$\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ca & cb & -c^2\end{vmatrix} = 4a^2b^2c^2$$

4. Find the inverse of the following matrix, using elementary transformations: $A = \begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}$.

5. For the following matrices A and B, verify that $(AB)' = B'A'$.

$A = \begin{bmatrix}1\\-4\\3\end{bmatrix}, B = [-1, 2, 1]$

6. Prove that: $\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \gamma+\alpha & \alpha+\beta\end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

7. Find $A^2 - 5A + 6I$, If $A = \begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$.

8. Using elementary row operations, find the inverse of the following matrix: $\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$

9. By using properties of determinant, prove the following:

$$\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix} = (a+b+c)^3$$

10. By using properties of determinant, show that: $\begin{vmatrix}1 & x & x^2\\ x^2 & 1 & x\\ x & x^2 & 1\end{vmatrix} = (1-x^3)^2$

11. Find the inverse of $A = \begin{bmatrix}3 & -1\\ -4 & 1\end{bmatrix}$ using elementary transformations.

12. If $A = \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then find k so that $A^2 = kA - 2I$. **[Ans. 1]**

13. By using properties of determinant, show that:

$$\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix} = (1+a^2+b^2)^3$$

14. By using properties of determinant, show that:

$$\begin{vmatrix}a^2+1 & ab & ac\\ ab & b^2+1 & bc\\ ca & cb & c^2+1\end{vmatrix} = 1+a^2+b^2+c^2$$

15. If $A = \begin{bmatrix}3 & -4\\ 7 & 8\end{bmatrix}$, show that $A - A^T$ is a skew symmetric matrix where A^T is the transpose of matrix A.

16. Using properties of determinants, prove the following:

$$\begin{vmatrix}x+4 & 2x & 2x\\ 2x & x+4 & 2x\\ 2x & 2x & x+4\end{vmatrix} = (5x+4)(4-x)^2$$

17. Let $A = \begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $B = \begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ and $C = \begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$, Find a matrix D such that $CD - AB = O$.

18. If a, b and c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, show that either a + b + c = 0 or a = b = c.

19. Prove that: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

20. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2 × 2 identify matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

21. Prove $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$

22. Show that: $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in AP.

23. (i) Prove that the sum of two skew-symmetric matrices is a skew-symmetric matrix.

(ii) Express the following matrix as the sum of a symmetric and a skew-symmetric matrix.

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

24. Prove that: $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$.

25. Find the matrix X so that

$$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

26. Using properties of determinants, prove the following:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

27. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

28. Express the following matrix as the sum of a symmetric and skew symmetric matrix and verify your result.

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

29. Without expanding, show that:

$$\Delta = \begin{vmatrix} cosec^2\theta & cot^2\theta & 1 \\ cot^2\theta & cose^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0$$

30. Solve the equation $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$

31. Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

32. Using properties of determinant, solve the following for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

33. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A^2 - 4A + 7I = 0$. Using this result calculate A^5.

34. Using properties of determinant, solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

35. Using property of determinant, prove the following:

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$$

36. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, express A as a sum of two matrices such that one is symmetric and other is skew symmetric.

37. By using properties of determinant, prove the following:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda+x)^2$$

38. Using properties of determinant, prove the following:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+4y & 8x & 3x \end{vmatrix} = x^3$$

39. Using properties of determinant, prove the following:

$$\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$$

40. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, then show that:

$$I + A = (I - A).\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

41. Prove the following using properties of determinant:

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$$

42. If a, b, c are positive and unequal, then show that the following determinant is negative:

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

43. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$, $n \in N$.

44. If a, b and c are all positive and distinct, show that

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ has a negative value.

45. Prove the following using properties of determinants:

$$\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$$

46. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then show that F(x), F(y) = F(x + y).

47. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

48. Prove, using properties of determinants:

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

49. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2$

50. Using properties of determinants, prove that:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \text{ Or } \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

51. Show that points A (a, b + c), B(b, c + a) and C(c, a + b) are collinear.

52. Solve the following system of equations by using matrix method.

$\frac{2}{x} + \frac{3}{y} + \frac{1}{z} = 4,\ \frac{4}{x} + \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} + \frac{20}{z} = 2$ **[Ans. x = 2, y = 3, z = 5]**

53. Find equation of line joining (1 , 2) and (3, 6) using determinants.

54. Using matrices, solve the following system of equations:

$x - y + z = 4;\quad 2x + y - 3z = 0;\quad x + y + z = 2$ **[Ans. x = 2, y = -1, z = 1]**

55. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

56. Using properties of determinant, prove the following:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$$

57. Find the area of the triangle with vertices at the points (2 , 7), (1, 1), (10, 8)

58. If $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1} and hence solve the following system of linear equations:

$2x + 3y + 11 = 0,\ -3x + 2y - 4z = 4,\ 5x - 4y - 2z = -9$ **[Ans. x = 1, y = 2, z = 3]**

59. Find the value of k, if area of triangle is 4 sq. unit when its vertices are (k, 0), (4, 0) and (0, 2)

60. Find the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

61. Using properties of determinant, show that:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

62. Using properties of determinant, prove that:

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

63. Using matrix, solve the following system of linear equations:

$x - y + 2z = 7;\ 3x + 4y - 5z = -5;\ 2x - y + 3z = 12$

64. Using properties of determinant, prove the following:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

LONG ANSWER QUESTIONS [6 MARKS]

1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, then $n \in N$.
2. If A, B are square matrices of the same order, then prove that adj. (AB) = (adj B) (adj A).
3. Without expanding evaluate the determinant:

   $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x + a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y + a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z + a^{-z})^2 & 1 \end{vmatrix}$, where $a > 0$ and $x, y, z \in R$.
4. Prove that every square matrix can be uniquely expressed as the sum of symmetric matrix and skew-symmetric matrix.
5. A mixture is to be made of three foods A, B, C. The three foods A, B, C contain nutrients P, Q, R as shown below:

Food	Grams per kg of nutrient		
	P	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

   How to form a mixture which will have 8 grams of P, 5 grams of Q and 7 grams of R?
6. If a, b, c are real numbers, then prove that

   $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$
7. Show that the matrix B^T, AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.
8. Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k if D(k, 0) is a point such that the area of ΔABD is 3 sq. units.
9. In a triangle ABC, if $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$, then prove that ΔABC, is an isosceles triangle.
10. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, then verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}.
11. Prove the following by the principle of mathematical induction:

    If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n.
12. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then find $\lim_{t \to 0} \frac{f(t)}{t^2}$
13. Find the matrix 'X' so that

    $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$